Enterprise Ontology

Contents

1. Existential Articulations

1.1 Directed graphs and incidence

We wish to formulate an existential articulation for an enterprise based on the use of directed graphs. See Appendix A for the formal mathematical definitions used.

The idea is that the vertices of a graph refer to *atomic*[i] event states and that the edges refer to *atomic* processes; atomic in the sense that they are viewed as indecomposable with respect to some intermediate event. Paths in such a graph also represent processes, but those that have intermediate states and are built up from those which are atomic. Thus a path should be interpreted as an *activity chain*[ii] which is punctuated by named event states and built up from atomic processes.

With this point of view, we introduce the fundamental graph, E_0, which represents all the events and processes that we are able to refer to. We will sometimes call this graph the *context graph*[iii] of the enterprise or simply the context. It should be viewed as our named 'reality', describing the finest level of detail that we allow our articulation to refer to.

Thus, when we wish to describe the behaviour of an enterprise or a construct within the enterprise, we will do this in terms of a subgraph[iv] of E_0. In this sense, a subgraph can be viewed as a construct that enumerates sets of paths, or processes, within E_0 that can be performed by the construct being described. Such a subgraph contains a great deal of information. It has within it a set of terminal vertices or states which are vertices of E_0 and which we say can be achieved by the construct being described. A subgraph also has the paths or activity chains that can reach these terminal states as well as the earliest initial states along which such paths may start.

Thus within an existential articulation we will associate subgraphs to constructs of interest which will encode a whole raft of information. For instance, at one level of a stratified existential articulation one has (service) products which we will model, and which produce from this model a subgraph of E_0. Intuitively, the (service) products in an existential articulation are the things that the enterprise can produce, the terminal states which can be reached. Implicit in this information is the fact that these terminal states may be arrived at and hence the way in which they are reached. The subgraph makes more explicit this latter point of view, which encompasses the former intuition.

1.2 Levels and subgraphs

The definition, or formalisation, of an existential articulation is as follows:

Definition 1.2.1 *An existential articulation*[v]*, E, is a tuple,*

$$\left(E_0, E_{1,1}, \ldots\ldots, E_{1,r_1}, E_{2,1}, \ldots, E_{2,r_2}, E_{3,1}, \ldots, E_{i,j}, \ldots, E_{n,1}, \ldots, E_{n,r_n}\right),$$

where E_0 is a directed graph, each $E_{1,k}$ is a subgraph of E_0 and in general, $E_{i+1,j}$ is a subset of $\left\{E_{i,1}, \ldots., E_{i,r_i}\right\}$

The idea here is that we have an existential articulation which has levels. By a level we intend all those things which comprise it,

Definition 1.2.2 *In an existential articulation, E, the i^{th} level*[vi] *is defined to be*

$$E^i = \left\{E_{i,1}, E_{i,2}, \ldots, E_{i,r_i}\right\}.$$

The i^{th} level contains r_i elements or constituents. As i increases we say that we move to a higher level and as i decreases to a lower one.

These levels should not be confused with the stratification of the enterprise although both describe increasing levels of complexity.

Each construct, $E_{i,j}$, defined above has a double subscript with each element a natural number. The first of these is an indication of the level at which the construct occurs and hence the ordering derived from the natural numbers has some meaning. In the second of these subscripts however ordering is not important and the natural numbers are used here as a convenient indexing set and no more.

Our main intention is that the constructs, $E_{i,j}$, are interpreted as *named* constructs[vii] within the enterprise which, built up from constructs from a level of complexity directly below, name co-ordinations of behaviour within the enterprise expressed in terms of E_0.

The directed graph E_0 represents the possible events and processes to which we are able to refer. Each subgraph in the first level therefore represents a *co-ordination of behaviour*[viii] that can be used in the enterprise. Each construct in the higher levels represent a construct within the enterprise that utilises or co-ordinates the constructs on the level below.

Thus with this formalisation, the levels above the first are co-ordinations of co-ordinations[ix] (C^2). We should stress that these constructs, $E_{i,j}$, are *not* all sets of subgraphs. In fact they are sets of sets of sets of sets ..... of subgraphs. Their meaning is best understood through the $\overline{E_{i,j}}$ graphs (Definition 1.2.5) which describe behaviour, although this ignores the way in which they are built up from the constructs at lower levels.

Suppose that one were to model an enterprise in this fashion, one may have a person in the enterprise who is able to repair all the different types of machine within the enterprise. One might represent this man in the first level as the subgraph of E_0 consisting of every edge in E_0 that is the repairing of a machine (and perhaps also the paths in E_0 that also correspond to such repairs).

At the next level we may want to collect people into workgroups, which utilise all the skills of the people within them and so are represented as subsets of the level below. Note that our point of view is that these constructs are indecomposable in the following sense. The repair man who forms part of a workgroup might only be used to repair certain types of machine. However, he retains the ability to repair all the machines in the company and so does the workgroup. If one wanted to represent the fact that the workgroup used the repair man in only one capacity one would make changes at the lower level. That is, instead of having a single graph representing the repair man's capabilities one would have multiple graphs corresponding to different uses of his skills.

1.2.1 Complexity

Given this structure it is useful to have names for the different sorts of complexity arising. Firstly the number of things in the first level clearly have a bearing on the rest of the articulation since any $E_{i,j}$ must indirectly refer to something on the first level. Although it is tempting to say that the atomic distinctions in E_0 are the ones that matter, it is the first level on which named constructs must be based. The atomic distinctions in E_0 need not all be used by the articulation and may in fact be redundant from the point of view of the enterprise (see reduction in section 1.5.1). Moreover, if a distinction in E_0 is not used on the first level, it cannot be referred to at any other level. Thus the number of things on the first level are a measure of the degrees of distinction that are induced by the articulation.

Definition 1.2.3 *In an existential articulation, complexity of the first kind*[x] *is defined to be the number of elements in the first level,* r_1.

The other sort of complexity arises in the number of levels themselves. This sort of complexity represents an effect of multiple levels of co-ordination, since the greater number of levels in an articulation, the greater the number of layers of constructs that need to be passed through in order that a co-ordination at the top be realised.

Definition 1.2.4 *In an existential articulation, E, the complexity of the second kind*[xi] *is the number of levels, n.*

1.2.2 Behaviour graphs

As promised we will describe the behaviour of a construct[xii] within the enterprise as a subgraph of E_0. First note that the union of a set of subgraphs of a common graph is also a subgraph. Thus for the second level, to describe the things it can do we take the union of all the subgraphs of E_0 that comprise it and denote this by a bar.

$$\overline{E_{2,i}} = \bigcup R_{2,i} = \bigcup_{E_{1,j} \in E_{2,i}} E_{1,j}$$

We proceed with the notion that if we have a construct which is itself a set of constructs then the large construct can co-ordinate the behaviours of its members and is able to do what that set can collectively do. One can extend this definition to the first level by putting $\overline{E_{1,j}} = E_{1,j}$. In other words we get the following definition,

Definition 1.2.5 *Let E be an existential articulation. Then* $\overline{E_{1,j}} = E_{1,j}$*, and inductively for* $i \geq 2$,

$$\overline{E_{i,j}} = \bigcup E_{i,j} = \bigcup_{E_{i-1,k} \in E_{i,j}} E_{i-1,k}$$

Thus each $\overline{E_{1,j}}$ is a subgraph of E_0 in which the paths describe the processes possible by $E_{i,j}$. The sense in which this existential articulation describes what can be achieved by the enterprise is encoded by these $\overline{E_{1,j}}$ graphs, although one should be aware that the graphs mean different things. Just as each $E_{i,j}$ is a named construct within the enterprise, the graph $\overline{E_{1,j}}$ describes what can be achieved by that particular construct.

Thus if one were to ask, "What can be achieved by the enterprise as a whole?", the answer would not necessarily be straightforward. Intuitively, the answer should be that the construct at the highest level co-ordinates the behaviour of the enterprise as a whole and so the graph corresponding to it is the description of what the enterprise may achieve. However, it is not necessarily the case that the top level will contain a single construct and so there may not be a canonical choice of overall co-ordination. Rather, multiple elements in the top level should be interpreted as there being different possible co-ordinations which are able to achieve different if overlapping things, and between which there is no specified co-ordination. This may happen in an enterprise that has multiple branches, each of which may draw upon some common set of resources, but are controlled by different entities that are autonomous with respect to each other.

Thus the graphs, $\overline{E_{1,j}}$, would tell one what each of these entities at the top level were able to achieve; and one could indeed deduce what could be achieved were they to work in concert. The point here is that if they are separate at the top level of the articulation, the interpretation intended is that there is no named way for them to work together or be co-ordinated. If such a mechanism did indeed exist it would appear at the next level up as a set which had as its elements all of the named entities that it could co-ordinate.

1.3 Constraints

It is not immediately apparent that an existential articulation as formulated above allows one to describe constraints. Suppose that some construct within a workgroup utilises some set of capabilities but that one wants to say that in its use of these capabilities it is constrained in its actions. The above definition seems to allow no scope for this as the $\overline{E_{1,j}}$ graphs describe *everything* that the named construct is able to do when co-ordinating its components, inductively down to the lowest level.

Our point of view here is that one can only make those distinctions that have been named. So suppose we have an existential articulation set up and one then wants to introduce constraints on some construct, which we shall call $e = E_{i,j}$, that describes some limiting of behaviour. In essence what one is doing is taking the constructs that comprise e, either at the level directly below or at levels below that, and saying that these constructs, (say capabilities or workgroups), can be distinguished in their behaviour so as to allow the constraining of e.

Our example above of the repair man is the case to bear in mind. This repair man can be described by either one subgraph that encompasses the totality of his behaviour or as multiple subgraphs which describe more finely the things he may achieve (or both). Suppose that one wanted to articulate a constraint that had to do with only using the repair man in specific capacities but one only had a description of the repair man as a 'large' graph and not the multiple subgraphs of it.

Then we would say that the existential articulation as it stands is not fine enough to make the distinctions needed to express the constraint. To refine the articulation one needs to add a set of subgraphs that correspond to using the repair man in specific capacities so that these capacities may be differentiated. In a certain sense the graph describing the repair man would be *cut* in certain ways to introduce subgraphs. The question of whether to retain the large subgraph corresponding to the repair man in his joined up capacity and at what level would have to be made according to the precise considerations in question. In effect one would be re-formulating the existential articulation in order to allow for finer distinctions and one may or may not want to retain some of the coarser distinctions made.

With this interpretation, if one wishes to introduce a constraint or some set of finer distinctions into an existential articulation, one can ask at what levels cuts need to be made in order to accommodate these distinctions. Of course, every time we introduce further distinctions, we at least increase the complexity of the second kind, and may have to increase the complexity of the first kind. This question is interesting in the sense that it asks what sort of distinctions one is making; is one dividing the separate tasks of some committee or is one partitioning the uses of some machine tool?

1.4 Levels and Complexity

The way that levels are set up leads one to the intuition that they represent different layers of complexity, so that a construct at a higher level can achieve more than another at some lower level. This is not entirely supported by the mathematics.

One might articulate the situation where some capabilities, say, were represented as subgraphs of E_0 at level one, in such a way that some were subgraphs of others. For instance, one might have a subgraph for our familiar repair man and also want to include the subgraphs for each of his individual skills. This would represent the information that we could call on the repair man in two different capacities; one as a man with a whole host of skills and one as several instances of his skill specific uses.

One possible confusion that may arise is in the existence of dummy levels. It is entirely possible that a level consists of sets each of which is a singleton, in the sense that each has only one element. We particularly concentrate on the case where there is such a set for each element of the level below.

At such a dummy level the behaviour of the enterprise would be no different to the behaviour of the same enterprise at the level below. For instance, if one wanted to look at the customer situations for an enterprise within an existential articulation, one may have that the enterprise does not ask this question, saying, "we do what we do". This fact would be represented by a dummy level. It should be emphasized that a dummy level does not over-determine the behaviour of an enterprise and neither does an enterprise where the top level is a set with only one element. In each case one can associate (at least one) subgraph of E_0 to describe the behaviour of the enterprise and such a subgraph might well have multiple paths between different states.

1.4.1 Derived Constructs

One can make explicit some of the inherent structure in an existential articulation. To start with, we have laid out how to take an element of a level and associate a subgraph to it that describes its behaviour. This expresses said element in terms of behaviour at the zeroth level. One may similarly stop the same process along the way and associate to the construct a subset of any intermediate level.

This can perhaps be seen more easily if we define the *naming dag*[xiii] of an existential articulation.

Definition 1.4.1 *If E is an existential articulation then the naming dag of E, denoted N_E, is the dag induced by the incidence structure of E at all levels from the first and above. That is, the vertices of N_E are $E_{i,j}$ and there is a directed edge from $E_{i,j}$ to $E_{i+1,k}$ if $E_{i,j} \in E_{i+1,k}$.*

Thus N_E is a simplicial directed graph which encodes the convention of naming with respect to inclusion.

The vertices of N_E are the named constructs within our enterprise and N_E is layered as E. Starting with some $E_{i,j}$ one can then look in N_E to find all the vertices which are connected to $E_{i,j}$ by an edge directed toward $E_{i,j}$. These are vertices at distance one below $E_{i,j}$ and form a subset of the level below, E_{i-1}.

One can clearly carry on this process and look at all the vertices of N_E which are at distance two below our chosen $E_{i,j}$ and form a subset of E_{i-2} associated to $E_{i,j}$. This subset will be all the things that are used by $E_{i,j}$ two levels below. In fact this construction may be made inductively so that one may specify which constructs, at any given level below i are used by $E_{i,j}$ and is equivalent to taking repeated unions. This can be used to assess the complexity of any $E_{i,j}$ or in fact any subset of E_i with respect to any lower level. One can also perform Q-analysis in this way as in Section 1.6.

Other objects of interest include the subgraph of E_0 that describes all the possible behaviour of the enterprise.

Definition 1.4.2 *The possible tactics*[xiv] *for an enterprise are given by the graph*

$$\overline{E} = E_{1,1} \cup E_{1,2} \cup \ldots \cup E_{1,r_1} = \bigcup E_1 = \bigcup \overline{E_{i,j}}.$$

It is clear that $\overline{E}$ is a subgraph of E_0 and that every $\overline{E_{i,j}}$ is a subgraph of $\overline{E}$. This graph thus gives a repertoire of possible tactics, but attention should be paid to the fact that $\overline{E}$ is in fact the union of all the graphs at the first level.

Continuing in the same vein, if we say that strategy is defined by what the enterprise ignores, then we can define,

Definition 1.4.3 *The strategy of an enterprise*[xv]*, σ_E, articulation is the largest subgraph contained within E_0 - $\overline{E}$.*

We note that $\overline{E}$ does *not* describe all the things that the enterprise can do but merely all the things that it might do, depending on how all its capabilities were co-ordinated. Also, the strategy σ_E is relative to E_0, being defined not in terms of the things that the enterprise does not do, but in terms of those behaviours for which it has no co-ordinations.

This last idea can in fact be also applied to any named construct within the enterprise.

Definition 1.4.4 *The strategy of an $E_{i,j}$, called $\sigma_{E_{i,j}}$, is defined to be the largest subgraph of E_0 contained in E_0 - $\overline{E_{i,j}}$.*

Also for any set of constructs within an existential articulation we can define their strategy as the intersection of the strategies of each individual construct. This will be the set of things that they collectively ignore.

Definition 1.4.5 *The strategy of a set of constructs $E_{i_1,j_1},\ldots,E_{i_k,j_k}$ in an existential articulation is defined to be $\bigcap E_{i_t,j_t}$.*

The strategy of the whole naming dag is then the strategy of all the constructs within it. One can then define the strategy dag,

Definition 1.4.6 *The strategy dag*[xvi] *of an existential articulation is the subgraph of the lattice of subgraphs of E_0 generated by the strategies of individual constructs, $\sigma_{E_{i,j}}$.*

Finally, we can define the ontology of behaviour of an enterprise:

Definition 1.4.7 *The ontology of behaviour*[xvii] *of an enterprise, $\exists_E$, is the structure defined by its strategy dag in relation to its $\overline{E}$.*

1.5 Enlarging and reducing the context

1.5.1 Changing E_0

It may be that after formulating an existential articulation, one then decides that one has made more distinctions than needed. It is therefore desirable to be able to change the context, E_0, of the articulation. This can be done in one of two ways.

Firstly, one could restrict E_0 to a subgraph to limit the context. This operation would forget the names of processes and events that were previously named. The whole existential articulation could correspondingly be limited and one would preserve the number of levels. However, it is quite conceivable that a number of dummy levels be created by such an operation.

Secondly, one could take a quotient of the graph E_0 in order to make atomic processes that previously were not atomic, perhaps even compressing them into event states. Again, one can extend this quotient to the whole articulation to preserve the number of levels but could create some redundancy, in the form of dummy levels. Once different elements of some levels may be identified in the new articulation.

We shall say that the articulation E is *reduced*[xviii] if either of the above operations is used to transform E_0 and therefore, E. We shall also say that E is reduced if a sequence of these operations is used. However, it should be noted that any sequence of reductions is equivalent to a single quotient followed by taking a subset (or vice versa).

An interesting question arises in this context. What is the largest reduction one can make to E_0 and therefore E that preserves the level structure or equivalently the naming dag N_E. This will always have an answer in terms of subgraphs of E_0 and in effect asks what distinctions are sufficient to support the existential articulation.

We also wish to have a notion of enlargement. We shall simply think of it as the reverse of reduction. So if E and E' are two existential articulations where E is a reduction of E', we shall say that E' is an *enlargement*[xix] of E.

Note that these definitions lack a certain symmetry in the sense that for reduction, one can deduce what happens to the whole articulation by knowing what happens to E_0. This is not the case for enlargement. If E' is an enlargement for E and we know the articulation E as well as the context graph for E', we cannot necessarily deduce the rest of the E' articulation. Given a finite E_0 for an existential articulation E we cannot find an enlargement. It should be given as extra data and then verified to be one. However, we could in principle enumerate the possibilities for an enlargement, although some external criteria would be needed to assess their appropriateness.

1.5.2 Changing levels

In changing the articulation we could, instead of concentrating on E_0, look at the levels. We only wish to look at compressing the levels.

Suppose that we have an existential articulation, E, and we wish to make the i^{th} and the $i\text{-}1^{th}$ levels into a single level. Recall that for each $E_{i,j} \in E_i$, we have $E_{i,j}$ is a subset of E_{i-1}. We then associate a subset, $E'_{i,j} = \bigcup E_{i,j} \subseteq E_{i-2}$, of E_{i-2} so that $E'_{i,j}$ behaves as if it were on the $i\text{-}1^{th}$ level. This is, it is the same as reducing the construct $E_{i,j}$ to the level directly below via the naming dag as described in Section 1.4.1. One could replace the existential articulation E with another where the levels i and $i\text{-}1$ have been amalgamated. We shall call any sequence of these amalgamations of levels a *compression*[xx] of the existential articulation. Such a compression makes less distinctions of complexity of the second kind (fewer levels) than its precursor, insisting that some number of constructs work at a more elementary level than before. This would be like a manager co-ordinating capabilities directly instead of via some workgroups. Compression, however, does not change the complexity of the first kind.

Similarly we shall say that if E' is compressed to E then that E' is an *expansion*[xxi] of E. So that in this situation, E' makes more distinctions of complexity (of the second kind) than E. As before, there is an asymmetry to this definition. If I know which levels are to be compressed in an existential articulation, E, I can deduce the result of the compression in terms of another existential articulation, E'. The same is *not* true of expansion. If I know which levels of E I wish to expand, I cannot deduce an expansion E' as in general there will be many possibilities. One therefore needs extra information, or in fact a candidate for the expansion which one can then verify.

1.5.3 Stratification

This leaves us in a position to relate the concepts of strata and levels. Intuitively there is a relation between the two, but it is not necessarily the case that they correspond. A stratification is a way of imposing a structure on an enterprise that relates to an Enterprise Ontology (see the definition for this in Section 3), in which conceptually similar constructs are grouped. A level says more about a layer of an enterprise, sets of which can be compressed to a stratum.

Definition 1.5.1 *A stratum*[xxii] *in an existential articulation, E, is a collection or set of some adjacent levels* $\{ E^i, E^{i+1},\ldots,E^{i+k}\}$.

Thus a stratum of an existential articulation, E, is in fact a level in an existential articulation E' where E' is a compression of E where some set of adjacent levels have been compressed. The set compressed to get E' would be the set corresponding to the stratum in E.

The existential strata[xxiii] normally referred to in an existential articulation are *capabilities*, being co-ordinations of events and processes; *workgroups*, being co-ordinations of capabilities; *(service) products*, being co-ordinations of workgroups; *business units*, being co-ordinations of (service) products; *propositions*, being co-ordinations of business units; and *customer situations*, being co-ordinations of propositions.

1.6 Q-analysis

Since a level is built up from the one immediately below, there is clearly an incidence structure present in an existential articulation. Namely if one wants to perform Q-analysis on one level *with respect to the one below*, one simply forms a level complex whose vertices are the elements of the lower level and whose simplices are the elements in the higher level.

The fact of inclusion gives one a way of associating to each element of the higher level a set of vertices, which form the corresponding simplex. Incidence of simplices is then induced by common sets of vertices.

The concepts of granularity and eccentricity[xxiv] are then straightforward to define but one should be aware that if one looks at the complex induced by a *whole* level one is not necessarily analysing a construct present in the enterprise. For instance if one were interested in the co-ordination properties of a workgroup, one would look at the capabilities within that workgroup and see how they interact. Thus, I think that in many situations one should take an element of a level, say a workgroup, and look at the way it is built up from the lower level - these being the simplices one should study. Incidence is then derived from the level yet lower down. This information being contained within the existential articulation or within the naming dag, N_E. The point being that the simplices one looks at correspond to a construct within the enterprise that attempts to co-ordinate them. Q-analysis can then demonstrate how difficult it is to co-ordinate these simplices.

One does not, however, have to restrict Q-analysis to looking at complexity between adjacent levels. One could, by using the procedure outlined in Section 1.4.1, reduce any construct at some high level to a set of constructs at any level below, and thus perform Q-analysis for this construct with respect to any level lower down. This will in fact be equivalent to performing a compression on the existential articulation and using Q-analysis on adjacent levels in the new articulation.

1.6.1 Holes

This abundance of simplicial complexes leads one to look at ways in which to measure available co-ordination in specific instances. We put forward the notion of *holes* as a means to this end.

A hole of the *first kind* is a ‘gap’ in the context graph, E_0, between a pair of vertices. This is a fundamental impossibility, in the reality described by E_0, of going from one place to another.

Definition 1.6.1 *If u and v are vertices of E_0 we say that there is a hole of the first kind[xxv] between u and v if there is no path in E_0 with initial vertex u and terminal vertex v. (see Appendix A for the definition of initial and terminal vertices of paths.)*

A hole of the first kind is insuperable within the context of E_0. In fact, E_0 is full of holes of the first kind since it is acyclic.

The other type of hole that we are interested in describes a lack of a suitable construct within a strategy dag to achieve a set of outcomes and processes.

Definition 1.6.2 *A subgraph H of E_0 is called a hole of the second kind[xxvi] if H is not a subgraph of any $\overline{E_{i,j}}$ for any $E_{i,j}$.*

A hole of the second kind may be observed in different ways. For instance, if one wanted to co-ordinate the behaviour of some set of constructs within the articulation, $E_{i_1,j_1},....,E_{i_k,j_k}$, we can turn the problem into one of looking for holes of the second kind. These constructs, $E_{i_1,j_1},....,E_{i_k,j_k}$, can be co-ordinated if and only if the graph $H = \overline{E_{i_1,j_1}} \cup .... \cup \overline{E_{i_k,j_k}}$, is a hole of the second kind.

This comes down to saying that the above construct can be co-ordinated if their combined behaviour is made possible by the behaviour of some ‘higher’ construct. Note that the higher construct is not required to match the combined behaviour of those lower down but just contain it. Also, with this process we can transform a zeroth level definition into one that is more level specific. One can look at which levels each construct appears, and at which levels certain holes may be ‘plugged’. This is done by simply keeping track of which subscripts appear in the $E_{i,j}$ that we are concerned with in any specific situation.

Any subgraph of the strategy will be a hole of the second kind since the naming dag does not refer to it. Similarly any subgraph of E_0 that meets the strategy non trivially will be a hole of the second kind. The more interesting holes appear as subgraphs of the repertoire of tactics, $\overline{E}$, since they represent a failure of co-ordination within the enterprise to *fully* implement the behaviour specified by the graph. In general an existential articulation will have many holes of the second kind, and can be viewed as missing vertices in the naming dag N_E in the sense that they point to a missing co-ordination.

1.7 Composition

1.7.1 Composition with a common context

Suppose that we have two existential articulations for enterprises

$E = (E_0, E_{1,1}, ..., E_{1,r_1}, ..., E_{n,1}, ..., E_{n,r_n})$ and

$F = (F_0, F_{1,1}, ..., F_{1,s_1}, ..., F_{m,1}, ..., F_{m,s_n})$

Then we want to be able to form the composition of these enterprises that describes the things that they can do together. However it should be noted that such a composition will not include any description of a co-ordination between the two different articulations since such a thing needs to be specified and cannot be deduced. In fact it is perfectly possible that such a co-ordination does not exist.

First we define a notion of *strong* composition[xxvii]. Suppose that in the articulations above E_0 is graph isomorphic to F_0 (ie they are the same graph) and $n = m$ so that they have the same number of levels. The composition of E and F is then an existential articulation whose zeroth level or context graph is $E_0 \cong F_0$. Then every level of the composite articulation is simply a union of the levels for the individual articulations. In other words, the first level for the composite articulation is $E_{1,1}, ..., E_{1,r_1}, F_{1,1}, ..., F_{1,s_1}$. In general the i^{th} level is $E_{i,1}, ..., E_{i,r_i}, F_{i,1}, ..., F_{i,s_i}$.

This definition makes two restrictions that we wish to avoid having to make in general. Firstly it assumes that they only talk about the same things. In fact we wish to be able to enlarge the language of each articulation so that they may at least have the ability to refer to the same things. Secondly, we wish to be able to compose naming dags that have a different number of levels. We achieve this second goal by introducing extra levels that carry no more information.

So suppose that we have two existential articulations E and F as above, but that $n < m$. Then we add a level to E with exactly r_n parts each of which is a singleton set containing a distinct member of the level below. Thus we have an articulation which carries the same information as E but which has one extra level. Proceeding in this fashion we may assume that E and F have the same number of levels. This in effect adds dummy levels[xxviii] to E, to express the fact that the questions that F asks in its articulation are wider than those addressed by E. E, however, does not grind to a halt in the face of these issues but replies by saying that the structures in place to deal with narrower issues will suffice for the broader issues also.

Secondly we wish to arrive at a common context graph. We take the view that one may identify common elements in each of E_0 and F_0, which can be realised as a possible empty graph D embedded into each of E_0 and F_0. Thus we may form the pushout $E_0 *_D F_0$ and amend E and F so that the zeroth level in each is this new graph. Clearly, this adds no new information to either context but simply enlarges the number of things that are not mentioned. Hence we may assume that E and F have the same zeroth level.

Thus to compose two articulations E and F we add redundant levels to ensure that they have the same number of levels and we change the context graph, via some identified common subgraph, so that they have the same graph at the bottom. Then we perform strong composition on the resulting constructs.

1.7.2 Changing the context

The composition outlined above assume that the existential articulations *E* and *F* are constructed in similar ways with both similar levels of detail taken into consideration and with similar layers of complexity considered.

This is clearly not ideal since the articulations may differ in both ways. We therefore propose the following way of modifying *E* and *F* in order to allow the composition above.

Firstly each of *E* and *F* should be enlarged as in Section 1.5.1, to refine the events and atomic processes that they are able to talk about. This should be done to some common level of detail so that it is possible to find an appropriate common subgraph *D* for E_0 and F_0. This overcomes the problem where, for instance, a much elucidated process in F_0 appears as an atomic process in E_0. This operation changes *E* and *F*, but we shall assume that the enlargement has been performed and retain their names.

Then we wish to expand each of the articulations *E* and *F* as in Section 1.5.2, so that they deal with the same levels of complexity. This will ensure that we do not get the situation where a single level in *F* is represented as multiple levels in *E*. Potentially, we would introduce expanded dummy levels into *F*, but the precise nature of the expansion would depend on our specific criteria for distinctions in complexity. Thus it may be that while the levels of complexity in *E* were not addressed in *F*, they still have some significance there and so would not necessarily lead to dummy levels.

However, we also wish to expand further so that we can compare the respective levels of *E* and *F*. So suppose that we have expanded *E* and *F* to contain the same distinctions in levels of complexity and that *E* had *n* levels while *F* had *m* levels. Then we expand *E* and *F* further, so that they are each articulations with at most $n+m$ levels. We intend this step to ensure agreement over the height of levels in *E* and *F*. The idea here is that after this second expansion, things on the i^{th} level of *E* intuitively correspond to things on the i^{th} level of *F*.

To recap, after the first expansion we ensure that multiple levels in *E* are not contained in a single level in *F*. The second expansion ensures that the ordering of the levels in each articulation is along the same principles. This second operation is likely to introduce dummy levels since it may be that the distinctions of complexity made by each articulation are different in nature. Hence to give them a common levelling will mean that each has nothing to say at the levels the other is interested in.

Although this expansion is described as taking place over two stages, it could clearly be performed in one. However, there are two distinct reasons for expansion and so it is more natural to consider the two stages.

After performing this enlargement and expansion for *E* and *F* we are in a position to compare like with like. The composition defined in the level above then makes sense. Thus we have a general way of composing two articulations.

Note, however, that in this general form, composition is *not* deduced. The enlargement and expansion are performed with respect to some external criteria of commonality. For the enlargement it is a question of common or at least sufficiently overlapping behavioural ontology. For the expansion, it is a question of common distinctions of complexity akin to common principles and detail in stratification. Both of these require some external validation to assess appropriateness.

1.7.3 Complexity and Composition

The notion of composition defined above essentially ensures that each part of the whole has the same number of levels in their individual articulation, and so each has the same level of complexity of the second kind after the enlargement and expansion have taken place. Note that since adding dummy levels is a form of expansion, we can assume that we move immediately on to the strong composition after refining respective articulations. Clearly the number of elements in the first level of the composite is the sum of the number of elements in each of the constituents (after enlargement and expansion), giving a simple relationship between complexities of the first kind in a composite construct.

However, as noted above, enlargement and expansion are not deductive processes and do require some external criteria for validation. Since expansion potentially affects the complexity of the first kind, so that it is not possible before refinements have taken place to find a simple relationship between the complexity of the composite and that of its parts. It is clear however, that expansion can only decrease complexity of the first kind.

Hence if I start with two existential articulations E and E' with complexity of the first kind r and r' respectively, then their composite will have complexity not more than $r + r'$. If no expansion takes place in this composition, or if expansions do not affect the first level then it will be exactly $r + r'$.

Since the 'join' of two sublattices within a larger lattice is fairly arbitrary in its outcome there is no simple relationship between the strategy dags of two articulations and their composite. However, if we look at the articulations *after* any enlargement and expansion, we can say that the overall strategy of the composite will be the intersection of the individual strategies of the parts when considered as subgraphs of their common pushout. This observation is fairly intuitive since it says that the things ignored by a composite are those things that are ignored by *both* its parts.

2. Referential Articulations

We wish to develop a formal framework in which to describe a stratified referential articulation[xxix] that mirrors the mathematical formulation of an existential articulation. The idea for the referential articulation is that vertices represent demand states and edges represent being driven from one demand state to another. When speaking of Enterprises, it is best to think of the existential articulation as describing all the things that the Enterprise can do; and the referential articulation as all the forms of demand that an Enterprise can identify in its environment. Thus it is possible for an Enterprise to identify multiple forms of demand, arising from different demand environments (demand situations), that do not have any demands in common.

As with the existential articulation, we will base the analysis on having a graph that is a particular articulation of a relation to satisfaction. The resulting graph, R_0, will describe every demand state and every instance of being driven that can be anticipated. We will of course introduce dynamism into the structure that will allow us to change R_0 if it's descriptive power is deemed inadequate to support the naming dag that anticipates its satisfaction.

With the existential articulation, we can assume direct access to the E_0 through observation, so that it is possible for the E_0 that results from a naming dag to be a subset of this 'directly observed' E_0. With the referential articulation this is not possible. The articulation of the R_0 can only be elaborated in support of the elaboration of the levels in its naming dag. This is because the drivers are only identified as such to the composite demand articulated by each named construct in the naming dag.

Each 'demand' in R_0 is a demand for satisfaction. The first level, broadly speaking, will consist of all the indecomposable anticipations of satisfaction that the naming dag is based on, expressed in terms of a subgraph in R_0. Each construct in the first level is therefore a way of describing an anticipation of the satisfaction[xxx] of a want defined by its constituent demands. As one moves up the levels, one has more complex anticipations of satisfaction, expressed in terms of these 'atomic' anticipations at level 1, which bring together larger subgraphs of demand expressed in terms of R_0.

Definition 2.0.1 *A referential articulation, R, is a tuple,*

$$\left(R_0, R_{1,1}, \ldots, R_{1,r_1}, R_{2,1}, \ldots, R_{2,r_2}, R_{3,1}, \ldots, R_{i,j}, \ldots, R_{n,1}, \ldots, R_{n,r_n}\right),$$

where R_0 is a directed graph, each $R_{1,k}$ is a subgraph of R_0 and in general, $R_{i+1,j}$ is a subset of $\{R_{i,1}, \ldots, R_{i,r_i}\}$

We note that this definition is the same as that for an existential articulation so that in mathematical terms an existential articulation has the same form as a referential articulation and vice versa. The point is that they have different interpretations. So we can form the same constructs as with the existential, but we need to form different meanings for them.

The idea of level is the same, except that whereas E_0 is a base, in relation to which the naming dag has height, R_0 Is a surface, with its naming dag defining deeper levels bringing together more and more complex anticipations of want.

Definition 2.0.2 *In a referential articulation, R, the i^{th} level is defined to be*

$$R^i = \{R_{i,1}, R_{i,2}, \ldots, R_{i,r_i}\}$$

The i^{th} level contains r_i elements or constituents. As i increases we say that we move to a deeper level and as i decreases to a shallower one.[xxxi]

With each construct $R_{i,j}$ we wish to associate a subgraph of R_0 which is the interpretation of the anticipations that $R_{i,j}$ is describing, at the level of the 'surface' organisation of demand.

Definition 2.0.3 *Let R be a referential articulation. Then, $\overline{R_{1,j}} = R_{1,j}$ and inductively for i≥2,*

$$\overline{R_{i,j}} = \bigcup R_{i,j} = \bigcup_{R_{i-1,k} \in R_{i,j}} R_{i-1,k}$$

A subgraph of R_0 encodes information about paths, initial vertices and terminal vertices. A path in R_0 will be a way of describing a whole network of demand states linked by drivers. Thus the bar graphs describe the *demand organisation* for each construct - the collection of ways that the construct can anticipate the satisfaction of a want via a `flow' of satisfaction across demand states. Each subgraph in the first level therefore represents an *anticipation of satisfaction* of demand. Each construct in the deeper levels represent an anticipation that is expressed in terms of the constructs in the level above it. Thus with this formalisation, the levels above the first are anticipations of anticipations (A^2)[xxxii].

2.0.1 Stratification

As with the existential articulation, the levels can be stratified, by ‘compressing’ intermediate levels within a strata. How are we to think about the levels in a referential articulation in terms of strata? [xxxiii]

The first stratum of the articulation will be a set of subgraphs that describe ‘*problems*’. The problems closest to R_0 are the ‘smallest’ problems from the point of view of anticipating the satisfaction of demands, although they may be very ‘large’ problems when it comes to defining behaviours that will satisfy them. The second stratum will be anticipations of problems occurring together that correspond to ‘*propositions*’. The third stratum will be ‘*value profiles*’ – the anticipation of a particular way of satisfying a set of propositions. The fourth stratum will then be the ‘*service requirements*’ that are anticipations of the satisfaction of these value profiles; the fifth will be the ‘*integrating processes*’ that are ways of anticipating the satisfaction of sets of these service requirements; and the sixth stratum will be the ‘*capability outcomes*’ that are anticipations of the satisfaction of the integrating processes. From the point of view of R_0, each of the constructs within each of these strata are describable in relation to the drivers that impact on them, and each of the constructs is also an organisation of demand. Thus the progressive elaboration of drivers and levels also becomes an elaboration of the relations within R_0.

One of the key issues to appreciate here is that as the articulation of levels goes deeper in the referential articulation, the constructs being identified are more likely to be directly supportable by behaviours at lower levels in the existential articulation; and vice versa. This is because a simple demand in its environment may require complex behaviours from the Enterprise, just as a simple behaviour may be able to `take part' in the satisfaction of highly complex configurations of demand.

2.1 The naming dag

The graphs, $\overline{R_{i,j}}$, show how to take each construct $R_{i,j}$ in the naming dag and associate a subgraph of R_0 to it. This is an inductive definition that reduces each level, one step at a time, to the level above (closer to the ‘surface’). If we stop this process on the way we can associate to any construct a set of constructs that it decomposes to on some higher level. This can be seen more easily if we look at the naming dag for the referential articulation. The naming dag will in fact list all the constructs in the referential articulation.

Definition 2.1.1 *If R is a referential articulation then the naming dag of R*[xxxiv]*, denoted N_R, is the dag induced by the incidence structure of R at all levels from the first and below. That is, the vertices of N_R are $R_{i,j}$ and there is a directed edge from $R_{i,j}$ to $R_{i+1,k}$ if $R_{i,j} \in R_{i+1,k}$.*

Just as we had tactics for an existential articulation, we have *demand organisation* for a referential articulation which would represent all the different ways that satisfaction can be anticipated by the enterprise. It is not necessary that all these ways are described by some construct $R_{i,j}$, but represents all the ways that satisfaction could be anticipated if one were able to utilise all the ways that they are anticipated on the first level.

Definition 2.1.2 *The possible demand organisation*[xxxv] *anticipated by an enterprise is given by the graph*

$$\overline{R} = R_{1,1} \cup R_{1,2} \cup ... \cup R_{1,r_1} = \bigcup R_1 = \bigcup \overline{R_{i,j}}.$$

Dually, we can define a demand situation addressed by an enterprise (referentially) to be the relation to all those demands that are not named by anything within the articulation - in effect a being driven by all the demands that are not addressed. In other words, we are thinking of a demand situation as a relationship to a driver context. This concept is clearly dependent on R_0.

Definition 2.1.3 *The demand situation*[xxxvi] *of a referential articulation, σ_R, is the largest subgraph contained within $R_0 - \overline{R}$.*

Similarly we could look at the things ignored by any construct within the articulation,

Definition 2.1.4 *The demand situation of an $R_{i,j}$, called $\sigma_{R_{i,j}}$, is defined to be the largest subgraph of R_0 contained in $R_0 - \overline{R_{i,j}}$.*

We can even look at the things ignored by a set of constructs within the articulation.

Definition 2.1.5 *The demand situation of a set of constructs $R_{i_1,j_1},....,R_{i_k,j_k}$ in a referential articulation is defined to be $\bigcap R_{i_t,j_t}$.*

The demand situation of the whole articulation is then the demand situation of all the constructs within it. One can then define the demand situation dag,

Definition 2.1.6 *The demand situation dag*[xxxvii] *of a referential articulation is the subgraph of the lattice of subgraphs of R_0 generated by the demand situations of individual constructs, $\sigma_{R_{i,j}}$.*

Finally, we can define the ontology of demand[xxxviii] for an enterprise:

Definition 1.4.7 *The ontology of demand of an enterprise, $\forall_R$, is the structure defined by its demand situation dag in relation to its $\overline{R}$.*

2.1.1 Complexity

As with an existential articulation we have notions for complexity of both the first and second kind. The definitions for these are the same as in the existential case. Briefly, complexity of the first kind is the number of constructs in the first (shallowest) level and complexity of the second kind is the number of levels (the depth)[xxxix].

The complexity of the first kind gives a measure of how many `surface level' problem distinctions we have made and which we may bring to bear in the statements of more complex demands.

The complexity of the second kind is a notion of how many layers of abstraction from the surface level we have made in formulating our most complex demands. Broadly speaking, a shallow level is one which is close to the directly experienced demands of the customer/client anticipated by the enterprise; and one which is deep is those forms of anticipation that are common to many different surface forms, and hence less directly attributable to ‘direct’ (surface) articulations of demand in the Enterprise's environment.

2.2 Changing the context

Since referential articulations are the same mathematically as existential articulations, we can perform the same operations on the structures. So we can reduce a referential articulation by taking subgraphs and quotients of R_0 and enlarge by doing the reverse.

A reduction of R_0 involves making less distinctions in demand states and drivers, and thus ignoring some demand states and drivers. Thus an enlargement involves refining the distinctions made and encompassing a greater repertoire of demands and drivers. Broadly speaking this corresponds to enlarging and reducing both forms of complexity of the naming dag, and changing the articulation to reflect this.

Similarly, we can compress levels by collapsing them and expand levels by doing the opposite. These operations involve a decrease or an increase in the complexity of the second kind that we are considering, but not in the complexity of the first kind.

Once again a stratum is then seen as a collection of adjacent levels and can be seen directly in a referential articulation that collapses these adjacent levels.

Definition 2.2.1 *A stratum in a referential articulation, R, is a collection or set of some adjacent levels $\{R^i, R^{i+1}, ..., R^{i+k}\}$.*

2.3 Q-analysis

The mechanics of Q-analysis is formally the same for referential articulations as for existential articulations. That is for any collection of constructs $R_{i,j}$ within the referential articulation we can form a simplicial complex of these constructs *with respect to any level that is shallower than all of them*. The concepts of eccentricity and granularity can then be used to analyse these complexes. It should always be noted that this analysis is relative to the shallower level that one picks as the base for the simplicial complex.

The notion of holes also translates exactly to the referential case. A hole of the first kind being a ‘gap’ in R_0 that represents the lack of a driver between two states of demand. It is this ‘real’ lack that is understood as *desire*[xl]. A hole of the second kind is an anticipation of want, realised as a subgraph of R_0, which is not encompassed by any of the graphs, $\overline{R_{i,j}}$. Thus a hole of the second kind is an anticipation of satisfaction that is not named in the naming dag. There may be many such holes, not all of which are interesting. In particular, any subgraph of the demand situation will be a hole of the second kind.

2.4 Composition

The definition of strong composition can be used and interpreted in the referential case. The meaning of such a composition is roughly as follows. If two anticipations of demand of an enterprise are composed referentially, the composite will bring their organisations of demand together in the sense that anything that it will anticipate as being wanted will be anything that either of the constituent organisations of demand anticipates being wanted.

In general, a composition of two referential articulations will not be appropriate until some common level of refinement has been established. Thus to compose two referential articulations we need to enlarge each until the atomic things they refer to have the same level of detail. Their complexity of the second kind then needs to be expanded so that their levels are comparable. The (strong) composition can then be applied to these new articulations. Just as the refinement in the existential starts by finding common events and processes at the lowest levels, and then work `up', with the referential, this search starts from common demands and drivers at the surface level, and works down towards deeper levels.

As with existential articulations, one can say something about the relationships between the complexity of composite referential articulations and their constituents. The complexity of the first kind cannot be greater in the composite than the sum of the complexities of the first kind in the constituents. In fact this complexity will be exactly equal to the sum where no expansion has taken place or where the expansion does not affect the first level. Also, as with existential articulations, the overall demand situation of a composite articulation is the intersection of the demand situations of its parts, after any enlargement and expansion.

3. Looking forward – next steps

So far we have seen how in taking a double articulation model for an enterprise it is possible to construct a mathematical formalisation based upon it. This formalisation allows us to make precise the description of certain types of behaviour and structural properties within an enterprise using the mathematical tools of graph theory and to a lesser extent category theory.

This approach has been successful in elaborating descriptions of both the existential and referential articulations with a strong emphasis on interpretive validity from a stratified viewpoint. In fact it has proved invaluable in the development of these ideas by the interaction of empiricism and formalism.

The existential articulation provides a description of the behavioural ontology of an enterprise or set of enterprises via certain types of graph. The referential articulation provides a description of the demand ontology – the organisation of demand in relation to an enterprise, again by the use of graphs that encode demand states and their drivers. The main goal for the next stage of the project is to form a composite existential and referential articulation, which describes how the behavioural and demand ontologies of an enterprise can be read as a common ontology – the *Enterprise Ontology*[xli]. This is bound to be an imperfect process in the sense that there will not be direct correspondences between all the different anticipations of satisfaction and behavioural co-ordinations that are available within an enterprise. The aim in forming a composite existential and referential articulation is precisely in highlighting these inadequacies or holes, and in describing or proposing deontic solutions to them. These deontic 'solutions' bring together anticipations of A^2's as A^3's in relation to the demand ontology, which are also C^3's in relation to the behavioural ontology[xlii], and which form the basis of a third articulation – the deontic articulation[xliii].

The potential correspondence between existential and referential articulations arises empirically by the identification of low level existential capabilities and their co-ordinations with the deep levels of anticipation in the referential articulation. Thus, for example, more complex demand situations are identified with simpler capabilities. In the referential articulation one might say that the demands supported by the use of a pen could take a whole host of complex and varied forms. In the existential articulation one would describe the various outcomes in which a pen could play a part. This ‘low to deep’ correspondence arises naturally due to the approach to defining the referential articulation; the point being that the referential is actually being formed as the *enterprise's own view* of the organisation of customer and client demand environments. Thus the existential and referential articulations are constructed with a view to supporting each other, so that the ‘low to deep’ composition is built into their structures.

An interesting question to ask is how different philosophies of construction would affect the final composition and what information would be sufficient to formulate the respective articulations. Our view is that if the referential were formulated by an external observer, rather than an internal one, the logic of the composition would remain the same but the problem of constructing a common enterprise ontology would be more difficult. As to the information needed to build an articulation, we do have rather inelegant mathematical conditions that allow the construction of E_0 (and respectively R_0) from a naming dag.

The problem arising is that a naming dag itself does not carry sufficient information to deduce anything about transitions between potential named states, but rather is part of the process of constructing an articulation. Thus, we intend to develop the mathematics in the context of articulation construction not as a means of providing a purely deductive process from some incomplete initial set of data, but rather as a tool for aid in the construction process. The mathematics would be used to highlight which questions needed an answer for a construction to be formed and the consequences of such answers.

The tools above would provide a sufficiently rich environment to construct the first layer of the deontic articulation. This articulation would describe the holes in the Enterprise Ontology arising from the composition of its Existential and Referential articulations. Issues such as under-determination[xliv] become key in the rationale behind the deontic articulation, whose existence is justified by the problems apparent in the composite existential and referential articulations and the extension of these problems.

Appendix A

Directed Graphs

Deffinition A.1

A *directed graph*[xlv] is a tuple (V,E,ι,τ) where
V is the vertex set
E is the edge set and

ι,τ are functions from E to V picking out the initial and terminal vertices of edges. By convention, ι and τ extend to $E \cup V$ by both acting as the identity map on V.

Note that in our definition E is *not* a relation on V (a subset of V^2). This is to allow us to consider graphs which have more than one edge between vertices.

A *subgraph* of a graph is intuitively a subset that is a graph with the inherited structure. This doesn't quite work, since a graph is a tuple but intuitively it is sound. Formally we would say that,

Definition A.2 *Let G =* (V,E,ι,τ) *be a graph. A subgraph of G is a tuple H =* (V′,E′,ι′,τ′)*$ where V′ $\subseteq$ V, E′ $\subseteq$ E$ and* ι',τ' *are the restrictions of* ι and τ *to the set E. Moreover H is also a graph, which is to say that the range of each of* ι' *and* τ' *is contained in V′.*

A *terminal vertex* of a subgraph is a vertex which is not the initial vertex of any edge in the subgraph. An *initial vertex* in the subgraph is not the terminal vertex of any edge in the subgraph.[xlvi]

A graph map, ϕ from the graph (V_1,E_1,ι_1,τ_1) to (V_2,E_2,ι_2,τ_2) is really a pair of maps from V_1 to V_2 and E_1 to $E_2 \cup V_2$ which are both denoted by ϕ, satisfying the condition that for every $e \in E_1$,

(i) $\iota_2(\phi e) = \phi(\iota_1 e)$$ and,

(ii) $\tau_2(\phi e) = \phi(\tau_1 e)$.

Additionally, the graph map between two graphs is called injective (resp. surjective) if it is injective (resp. surjective) on both the vertex and edge sets.

Our definition of graph map allows one to collapse edges to vertices and is more general than the one sometimes given.

A path[xlvii] in a graph G = (V,E,ι,τ) is a sequence of edges $(e_1,...,e_r)$ such that $\tau e_{i-1} = \iota e_i$ for $2 \leq i \leq r$. A path is called a cycle if in addition, $\tau e_r = e_i$. A path $(e_1,...,e_r)$ is said to have initial vertex ιe_i and terminal vertex τe_r.

A directed acyclic graph[xlviii] or *dag* is a directed graph that does not have any cycles.

Given any directed graph, G, we may define a relation ~ on $V \cup E$ in the following way. If e is an edge of G with $\iota e = u$ and $\tau e = v$ then we say that u ~ e, e ~ v and u ~ v. Taking the transitive, symmetric and reflexive closure of this relation we get an equivalence relation. The equivalence classes will be subgraphs of G, called components, and G is called *connected* if there is only one equivalence class.[xlix]

Glossary

i atomic events and processes
ii *Activity chain*
iii *context graph, E_0*
iv *subgraphs*
v *Existential Articulation*
vi *level*
vii *named* constructs
viii *co-ordinations of behaviour* (C^1)
ix co-ordinations of co-ordinations of behaviour (C^2)
x *complexity of the first kind*
xi *complexity of the second kind*
xii The *behaviour* of a construct - $\overline{E}$
xiii *naming dag, N_E*
xiv *tactics* of an enterprise, $\overline{E}$
xv *strategy* of an enterprise, σ_E
xvi *strategy dag*
xvii *ontology* of behaviour, $\exists_E$
xviii *reduction* of *E*
xix *enlargement* of *E*
xx *compression*
xxi *expansion*
xxii *stratification*
xxiii existential strata: *capabilities, workgroups, (service) products, business units, propositions, customer situations.*
xxiv *eccentricity* and *granularity* of strata
xxv *a hole of the first kind*
xxvi *a hole of the second kind*
xxvii *strong* composition
xxviii *dummy levels*
xxix *referential articulation*
xxx *anticipation of satisfaction* (A^1)
xxxi *depth* of levels in the naming dag for the referential articulation
xxxii *anticipations of anticipations of satisfaction* (A^2)
xxxiii referential strata: *problems, propositions, value profiles, service requirements, integrating processes, capability outcomes*
xxxiv *naming dag, N_R*
xxxv *demand organisation*
xxxvi *demand situation*
xxxvii *demand situation dag*
xxxviii *ontology of demand, $\forall_R$*
xxxix referential *complexity of the first and second kinds*
xl *desire* as the experience of a hole of the first kind
xli *Enterprise Ontology*
xlii A^3 and C^3
xliii *deontic articulation*
xliv *under-determination*
xlv *directed graph*
xlvi *initial* and *terminal* vertices
xlvii a *path* in a dag
xlviii *dag* – directed acyclic graph
xlix *connected* dag